M.Sc. - I (Mathematics) (NEP Pattern) Semester-I

NEP-64-1 - Numerical Analysis

P. Pages : 2 **GUG/S/25/15115**

Time : Three Hours Max. Marks : 80

Notes : 1. Solve all **five** questions.
2. Each question carries equal marks.

UNIT - I

1. a) Discuss the Regula Falsi method for finding the roots of equation. **8**

b) Show that the order of convergence of Newton's method is 2. **8**

OR

c) Let g(x) be continuous on [a, b], and assume $g([a,b]) \subset [a,b]$. Furthermore, assume there is a constant $0 < \lambda < 1$, with **8**

$|g(x) - g(y)| \leqq \lambda |x - y|$ for all $x, y \in [a, b]$

Then $x = g(x)$ has a unique solution α in [a, b]. Also, the iterates $x_n = g(x_{n-1})$, $n \geq 0$ will converge to a for any choice of x_0 in $[a, b]$.

d) Obtain the iteration formula of Muller's method for finding roots of polynomial. **8**

UNIT - II

2. a) Let $x_1, x_2, ..., x_n$ be distinct real numbers, and let f be a given real valued function with $n+1$ continuous derivatives on the interval $I_t = H\{t, x_0, x_1 ....., x_n\}$, with t some given real number. Then prove that there exist $\xi \in I_t$ with **8**

$$f(t) - \sum_{j=0}^{n} f(x_j) l_j(t) = \frac{(t - x_0)....(t - x_n)}{(n-1)!} f^{(n+1)}(\xi)$$

b) Prove that for $k \geq 0$ **8**

$$f[x_0, x_1, ..., x_k] = \frac{1}{k! h^k} \Delta^k f_0, \text{ where } f_0 = f(x_0) \& f_i = f(x_i)$$

OR

c) For any two functions f and g and for any two constant α and β, show that **8**

$$\Delta^r [\alpha f(x) + \beta g(x)] = \alpha \Delta^r f(x) + \beta \Delta^r g(x) r \geq 0$$

d) Find the Hermite interpolating polynomial for which **8**

$p(a) = f(a) \quad p'(a) = f'(a)$

$p(b) = f(b) \quad p'(b) = f'(b)$

UNIT - III

3. a) Obtain an intermediate polynomial approximation $r_1^*(x)$ for the function $f(x) = e^x$ on the interval $[1, -1]$ **8**

b) Discuss the Gram-Schmidt theorem. **8**

OR

c) Let $\{\phi_n(x)\} | n \geq 0\}$ be an orthogonal family of polynomials on (a, b) with weight function w(x). With such a family we always assume implicitly that degree $\phi_n = n \; n \geq 0$. If f(x) is a polynomial of degree m, then prove that **8**

$$f(x) = \sum_{n=0}^{m} \frac{(f, \phi_n)}{(\phi_n, \phi_n)} \phi_n(x)$$

d) Assuming $[a, b]$ is finite, then prove that **8**

$$\lim_{n \to \infty} \left\| f - r_n^* \right\|_2 = 0$$

UNIT - IV

4. a) Obtain Simpson three-eights rule of integration. **8**

b) Obtain simple trapezoidal rule with error. **8**

OR

c) Assume [a, b] is finite. Then prove that the error in Gaussian quadrature. **8**

Satisfies $E_n(f) = \int_a^b w(x) f(x) dx \sum_{j=1}^{n} w_j f(x_j)$

$$|E_n(f)| \leq 2\left[\int_a^b w(x) dx\right] \rho_{2n-1}(f) \; n \geq 1$$

with $\rho_{2n-1}(f)$ the minimax error from $\rho_n(f) = \underset{\deg(q) \leq n}{\text{Infimum}} \|f - q\|_\infty$

d) Obtain the expression for Peano-Kernel error formula. **8**

5. a) Show that the Newton's method for determining a square root of A has the form. **4**

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{A}{x_n}\right).$$

b) Obtain the expression for $p_2(x)$ by Lagrange's interpolation. **4**

c) State the Weierstrass theorem and Taylor's theorem. **4**

d) Define- **4**

i) Asymptotic error estimate ii) Degree of precision.

****